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THE DEGENERATE SUPERDENSE GAS OF ELEMENTARY PARTICLES

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The composition of a degenerate gas whose density is of the order of nuclear density or higher, is considered. The temperature is assumed so low that all types of fermions are degenerate. It is shown that, with increasing density, different hyperons should successively appear and increase in number. They should be stable because of the Pauli principle. The threshold densities of different hyperons are calculated. Paradoxically, the smallest threshold density does not correspond to the Λ-hyperon, having the smallest mass of rest, but to the Σ^- - hyperon.

In accordance with this, a sufficiently massive cosmic body in gravitational equilibrium should consist of a hyperon core, a neutron layer, and an outer envelope having the usual composition (electrons, protons, and composite nuclei).

1. INTRODUCTION

At the present time, the study of various schemes of stellar evolution is becoming increasingly important in astrophysics. Numerous attempts are being made to interrelate the various observed stellar states by means of evolutionary models. In some papers, evolutionary models are constructed for entire galaxies.

The main feature of most of the proposed models describing the origin and evolution of stars and galaxies is that their authors assume an initial hypothetical state in which the matter forming the star or galaxy is a tenuous gas. It is assumed that stars originate through the condensation of this tenuous gas, the stellar matter remaining a classical perfect gas throughout all of the initial phases of evolution.

However, the analysis of extensive observational material in the case of young stars, as well as young galaxies, leads to the conclusion that, in the course of formation of stellar groups and galaxies, evolution proceeds from dense protostellar bodies to states of lower density. In other words, very dense protostellar bodies give rise to more or less numerous groups of stars, together with a large quantity of diffuse interstellar matter. Thus, there emerges the hypothesis that the normal stellar and diffuse states of matter are preceded by a superdense state.

Three series of observational data can be used as evidence in favor of the superdense initial state of matter.

The first series deals with galaxies and groups of galaxies. These data have been analyzed in [1]. It seems to us that it is the universe of galaxies which provides the most direct indication that evolution proceeds in the direction of decreasing densities. As has been mentioned in [1], there are data that provide evidence that galaxies and spiral arms are formed from matter originally contained in the nuclei of galaxies. These nuclei have small dimensions and a high density. Inasmuch as the stellar systems born in this manner cannot be formed from stars belonging to the normal type of stellar population found in the nuclei, we must assume that the nuclei may contain appreciable quantities of protostellar matter.

Another series of data refers to the origin of stellar groups forming stellar associations. As has been pointed out in [2], the presence of close groups of stars and Trapezium-type systems in associations and, in particular, inside the central regions of large gaseous nebulae found in O-associations, provides evidence against the view that stellar associations are formed from diffuse nebulae. The properties of Trapezium-type systems indicate that they arose from the division of a massive and very dense body.

The phenomenon of flares in UV Ceti stars, as well as in many of the T-association members, should be interpreted as a process of energy liberation, the energy being brought to the surface from the inner regions of the star in large but discrete amounts. It is reasonable to assume that this represents the eruption from the interior of quantities of protostellar matter still remaining in the central regions of the star. In this case, the energy of flares can be interpreted as the energy of transition from the protostellar to the normal state of matter [3].

Of course, one cannot assume that the initial state is the same in all cases mentioned above. It is quite possible that protostellar bodies giving rise to entire galaxies are markedly different from those giving rise to stellar associations, or simple systems of the Trapezium type. Moreover, it should be realized that the argument in favor of the existence of very dense protostellar states of matter is still not completely conclusive or final. However, it is, nevertheless, sufficiently strong to encourage us to study the possibility of the existence of bodies with cosmic dimensions in superdense states, in particular, in states with densities of the order of nuclear densities or higher, and to determine the properties of matter under these conditions.

The conclusion on the possible existence of superdense states of neutron stars was reached some time ago by Zwicky [4,5], who attempted to explain supernova explosions. The corresponding theoretical model was developed by Oppenheimer and Volkoff [6]. At the same time, it should be noted that, much earlier, L. Landau [7] speculated on the possibility of superdense cores in massive stars.

It is to be expected that superdense states, in general, possess very complex properties. Therefore, it is desirable to consider first of all the states for which the absolute temperature is close to zero or, more accurately, states with a temperature sufficiently low so that all types of fermions form a degenerate gas everywhere in the star. At the same time, it is also desirable to study, in addition to stable and equilibrium states, the possibility of the existence of metastable states which correspond to higher values of stellar energy.

This will allow us to consider, at the first stage of the investigation, the processes leading to the liberation of energy during the transition from a metastable state to a stable one.

The most important property of superdense states, as will be seen from the following, must be the presence of hyperons in the star, in addition to neutrons. Since, at sufficiently low temperatures, the nucleon (neutron and proton) gas will be highly degenerate, hyperons with energies lower than a threshold value will become stable, because the nucleons arising from their decay cannot find a place in phase space in accordance with the Pauli principle. For the same reason, the interconversion of different types of hyperons is also impossible. The hyperons present must also form a degenerate gas. In the following section, we will investigate the properties of a neutron-hyperon gas, or, more briefly, of a baryon gas at $T = 0$.

2. THE BARYON GAS

Let us consider the behavior of stellar matter when the density is of the order of nuclear density, or higher, at a temperature T = 0. Under these physical conditions, the nucleon and electron gas will be completely degenerate. At densities above a certain value, the threshold Fermi energies for nucleons and electrons become so high that it is more economical of energy if some of the matter changes from the nucleon state into the hyperon state.

Under the physical conditions of interest to us, it is strictly speaking, immaterial at equilibrium which of the elementary particles under consideration are primary, and which are secondary, i.e., those that have arisen from the primary ones. Nevertheless, for convenience, we consider in this section that nucleons are the primary particles from which, as the result of definite processes, hyperons can be formed.

At this stage, we are not interested in the precise specification of the process responsible for the formation of the superdense state of matter. It is possible to assume that the given superdense state was formed from one of still higher density. In this case, there must have been a conversion of heavy baryons into lighter ones. The reverse process of formation of superdense matter from a state of lower density can also be hypothesized (although we consider this to be unlikely). In this case, some of the nucleons should have changed into hyperons. In the first case, where the heavy baryons are converted into lighter ones, the process can take place in several ways (π-meson emission or leptonic transitions), while, in the second case, the conversion must take place almost exclusively by means of leptonic transitions if the temperature remains low.

When speaking of leptonic transitions, we have in mind the following elementary processes [8,9]:

$$\begin{aligned} n &\to p + e^- + \bar{\nu}; & p + e^- &\to n + \nu, \\ \Lambda &\to p + e^- + \bar{\nu}; & p + e^- &\to \Lambda + \nu, \\ \Sigma^\circ &\to p + e^- + \bar{\nu}; & p + e^- &\to \Sigma^\circ + \nu, \\ \Sigma^- &\to n + e^- + \bar{\nu}; & n + e^- &\to \Sigma^- + \nu, \\ \Xi^- &\to \Lambda + e^- + \bar{\nu}; & \Lambda + e^- &\to \Xi^- + \nu, \\ \Xi &\to \Sigma^\circ + e^- + \bar{\nu}; & \Sigma^\circ + e^- &\to \Xi^- + \nu, \end{aligned} \tag{2.1}$$

where the symbols n, p, Λ, Σ, Ξ, e^-, ν and $\bar{\nu}$ denote the neutron, proton, Λ -, Σ -, Ξ - hyperons, the electron, neutrino, and antineutrino, respectively.

If the formation of the superdense matter from less dense matter could take place at extremely high temperatures, of the order of 10^{10} degrees with subsequent cooling, then π-mesons can also play a role in the formation of hyperons.

However, we repeat that, in the present paper, we are not interested in the way in which the superdense states are formed. Nevertheless, in order to determine the relative abundance of the various types of particles as a function of density, we will use the table of elementary processes given above to obtain relations between the chemical potentials of the elementary particles. At T = 0, these relations can be written as

$$\begin{aligned} \mu_{\Xi^\circ} &= \mu_{\Sigma^\circ} = \mu_\Lambda = \mu_n, \\ \mu_{\Sigma^+} &= \mu_p = \mu_n - \mu_e, \\ \mu_{\Xi^-} &= \mu_{\Sigma^-} = \mu_n + \mu_e, \end{aligned} \tag{2.2}$$

where the symbol μ has been used for the chemical potentials, or, in other words, the threshold energies of the corresponding particles. In (2.2) it has been assumed that the chemical potential of the neutrino is equal to zero. The vanishing of the chemical potential of the neutrino is due to the absence of these particles from the volume of the star*; in fact, these particles must escape from the star as soon as they are produced, without experiencing any interactions (the cross section for the interaction of neutrinos with electrons is of the order of 10^{-44} cm^2 [10]). The meaning of the above statement about the chemical potential of the neutrino becomes clear if we look at (2.4), which is valid for any particle obeying Fermi statistics at T = 0.

The threshold momentum for fermions is given by

$$p_k = (3\pi^2)^{1/3} hN^{1/3}, \tag{2.3}$$

*If the mass of the neutrino is very small but different from zero, then some of these particles may be present within the star. However, a small rest mass means that the number of particles present is negligibly small and need not be taken into account.

where the subscript k = e, n, Λ, etc., denotes the type of particle, $\hbar$ is Planck's constant divided by 2π, and N_k is the number of particles of the given type per unit volume. The chemical potential of the particles is equal to their threshold Fermi energy:

$$\mu_k = c\,[M_k^2 c^2 + (3\pi^2)^{2/3} \hbar^2 N_k^{2/3}]^{1/2}. \tag{2.4}$$

Equations (2.2) and (2.4), together with the condition that the substance be electrically neutral, completely determine the concentration of the various particles N_k as a function of the neutron concentration N_n. However, before we proceed to calculate these quantities, we would like to present another derivation of the equations obtained in this section.

3. THE CONCENTRATION OF VARIOUS PARTICLES IN A HIGHLY DEGENERATE BARYON GAS

In the preceding section, the problem of the relative concentration of the various components of matter at T = 0, and very high density, was solved by a consideration of the possible reactions between the elementary particles. However, it is obvious that the result should be independent of the exact form of the interactions and on the probabilities for the various elementary transformations.

Therefore, it is desirable to present a derivation of the equations determining the concentration of the various types of baryons at T = 0, based on a number of general principles. The following three principles provide a natural starting point for the discussion.

1. At equilibrium the energy of the system must be a minimum.

2. In all conceivable processes leading to the establishment of a state of statistical equilibrium between the various components of matter, the number of baryons must be conserved.

As is known, there is also a law of conservation of leptons in elementary processes. However, under conditions when the neutrinos and antineutrinos formed can comparatively freely escape from the star, the number of leptons in a given volume cannot be considered as fully specified. In this case, the number of leptons becomes determinate, and then, in a statistical sense, only as the result of the establishment of thermodynamical equilibrium, the parameter determining the total number of leptons being the given total number of baryons.

3. The star as a whole, as well as its separate macroscopic regions, must be electrically neutral.

Using these basic assumptions, we can determine the concentration of the various possible components of matter at very high densities and absolute zero. Under the physical conditions that interest us, the possible components of matter can be electrons, μ^-- and π^--mesons, protons, neutrons, Λ^-, Σ^0-, Σ^+- Σ^--, Ξ^--, Ξ^0-hyperons, as well as the excited nucleons p* and n*. By excited nucleons, we mean the isobars of the proton and neutron with $\tau = 3/2$ and $j = 3/2$, where τ and j are the isotopic spin and angular momentum, respectively. Very recently, the existence of another two isobaric states of the nucleon with higher excitation energies (approximately 750 and 1000 Mev) has been established experimentally. In the following, we will only consider the first isobars because extremely high densities of matter are required for the excitation of the higher isobaric states.

The existence of excited states of the known hyperons is also possible. However, even if such states exist, extremely high densities are necessary for their excitation.

Finally, the possibility that higher hyperons, i.e., hyperons with masses exceeding that of the Ξ^-- hyperon, will be discovered in the future, cannot be excluded. However, the appearance of these particles in a degenerate gas at low temperature is also only possible at exceptionally high densities. Consequently, when we consider matter with a density lower than a certain limiting value, these particles need not be taken into account. As regards the other elementary particles known at the present time, namely, positrons, photons, neutrinos, μ^+-, π^+-, π^0-, and K-mesons, all of these particles must be absent at a temperature equal to zero, since there is no reason which would prevent their decay or annihilation. The situation is completely different in the case of hyperons, excited nucleons, μ^--, and π^--mesons. On account of the complete degeneracy of the electron and nucleon gas (T = 0), and the action of the Pauli principle, these particles, at sufficiently high densities, become completely stable. This occurs because the decay products of the particles considered cannot find an unoccupied position in phase space, since the latter is completely filled up to a certain threshold value of the momentum. Each type of unstable particle has its own threshold density at which it becomes stable. All hyperons and nucleon isobars have half-integral spins and, therefore, the gases formed by them will also be highly degenerate, so that transitions between the various hyperon states will be forbidden. The stability of the π^--mesons is ensured by the presence of a highly degenerate gas of μ^--mesons.

It should be noted that a priori it is impossible to state which types of real leptons will be present at T = 0 for a given concentration of baryons. For a given density, it is only possible to say that, if one of the types of leptons is present, the antilepton conjugate to this type must be absent. Calculations show that electrons should be present with baryons (positrons with antibaryons). Consequently, positrons should be absent. As regards μ^-- and π^--mesons, their presence is unavoidable once the presence of an electron gas is assumed.

π^--Mesons occupy a special position under the physical conditions considered (high densities of matter, T=0). Since π^--mesons obey Bose-Einstein statistics, all of them will be in a state of lowest energy, i.e., they will be motionless. Thus, they form a highly degenerate Bose gas. It will be shown below that, for densities above the threshold for the creation of π^--mesons, the concentration of these particles is of the same order of magnitude as that of each of the baryons. As a result of this, the temperature of a degenerate π^--meson gas is found to be very high.

Thus, the energy contained in a unit volume of the medium is given by

$$\rho = \frac{c}{2\pi^2 h^3} \sum_k a_k \int_0^{p_k} p^2 (M_k^2 c^2 + p^2)^{1/2} dp + N_\pi \cdot m_\pi c^2, \tag{3.1}$$

where p_k is the threshold momentum for the kth particle. The subscript k runs through the values e, μ, p, p^*, n, n^*, Λ, Σ°, Σ^+, Σ^-, Ξ^-, and Ξ° for electrons, μ^--mesons, various forms of nucleons, and hyperons, respectively. N_π is the concentration of π^--mesons, and a_k is a constant factor which takes into account the number of possible spin states of the particles; $a_k = 2s + 1$, where s is the spin quantum number. For excited nucleons, $s = 3/2$, so that $a_k = 4$, while, for all of the other particles, $s = \frac{1}{2}$ and $a_k = 2$. In the case of hyperons, the value $s = \frac{1}{2}$ **cannot be** considered to be firmly established. The first attempts to confirm this value of the spin experimentally have been described at the Kiev Conference [9].

For the threshold momentum of fermions, we have

$$p_k = \left(\frac{6\pi^2}{a_k}\right)^{1/3} h N_k^{1/3}. \tag{3.2}$$

According to the first principle stated above, the concentrations of the various components of matter N_k should be such that, for a given density, the energy (3.1) is a minimum.

To find the energy minimum, we must adjust the values of the variables N_k. Before we do this, however, it is necessary to take into account the second and third principles which introduce certain restrictions on the variation of the variables N_k. Thus, the third principle, together with (3.2), reduces to the following equation:

$$p_p^3 + 2p_{p^*}^3 + p_{\Sigma^+}^3 - p_e^3 - p_{\Sigma^-}^3 - p_{\Xi^-}^3 - p_\mu^3 - 3\pi^2 h^3 N_\pi = 0, \tag{3.3}$$

while the second principle can be written as

$$\frac{1}{2} \sum_k a_k p_k^3 = \text{const}, \tag{3.4}$$

where, with a spherically symmetrical distribution of mass, the constant depends only on r, the distance from the center of the star. In (3.4), the summation is carried out over all baryons.

Thus, we have to determine the minimum of the function (3.1), together with the supplementary conditions (3.3) and (3.4). As is known, the problem in this case reduces to the determination of the minimum of the function

$$\Phi = \frac{c}{2\pi^2 h^3} \sum_k a_k \int_0^{p_k} p^2 (M_k^2 c^2 + p^2)^{1/2} dp + N_\pi \cdot m_\pi c^2 +$$
$$+ \alpha (p_p^3 + 2p_{p^*}^3 + p_{\Sigma^+}^3 - p_{\Sigma^-}^3 - p_{\Xi^-}^3 - p_e^3 - p_\mu^3 - 3\pi^2 h^3 N_\pi) + \tag{3.5}$$

(more)

(continued)

$$+\ \beta\,(p_n^3 + 2p_{n^*}^3 + p_\Lambda^3 + p_{\Sigma^\circ}^3 + p_p^3 + 2p_{p^*}^3 + p_{\Sigma^+}^3 + p_{\Sigma^-}^3 + p_{\Xi^-}^3 + p_{\Xi^\circ}^3), \tag{3.5}$$

where α and β are as yet undetermined parameters.

Equating to zero the derivatives of the function Φ with respect to the particle concentrations N_k (or, for convenience, in the case of fermions, it is possible to differentiate with respect to the threshold momenta p_k), we find the necessary conditions for the energy of the mass distribution to be a minimum. At this stage, however, we should pay attention to a mathematical refinement which is important physically. The fact is that the derivatives of the function Φ must be zero for the energy to be a minimum only when this minimum occurs for values of each of these variables lying within the allowed range of variation. If the minimum value is attained when one of the variables assumes a limiting value, then the derivative of Φ with respect to this variable need not necessarily be zero. Thus, for example, it was shown above that π^--mesons can only appear at very high densities. For baryon concentrations below the threshold for π^--mesons creation, the function Φ must have been a minimum for $N_\pi = 0$; whereas this condition does not follow from the equations obtained by equating the derivatives of Φ to zero.

Therefore, we should be very careful and use each of the equations given below only when we are satisfied that, for the given baryon density, the variable, with respect to which the function Φ has been differentiated, is different from zero. Because of this, the whole range of variation of baryon concentrations will be subdivided in the following into individual subintervals which differ from one another with respect to the relative concentrations of the elementary particles, and which will be called by us the "phases" of the gas.

We could, of course, obtain a separate function Φ for each phase, and then find its minimum value. However, it appears to us to be better to consider, first of all, the highest phase in which all of the particles listed above are present in the medium, i.e., to write down (3.5). Then if, in the equation describing the condition of equilibrium between the various components, we omit the parameters of all the particles that are absent from the phase that is of interest to us, we will automatically obtain all the necessary equations determining the concentrations of the particles for this phase.

In all cases, with the exception of the variable N_π, the differentiations with respect to the N_k can be replaced by a differentiation with respect to the corresponding threshold momentum p_k. Equating the derivatives of Φ to zero, we obtain the following equations:

$$\frac{\partial \Phi}{\partial p_k} = \frac{c}{\pi^2 h^3}\, p_k^2\,(M_k^2 c^2 + p_k^2)^{1/2} + 3\,(\alpha + \beta)\, p_k^2 = 0, \tag{3.6}$$

where $k = p,\ p^*$ and Σ^+,

$$\frac{\partial \Phi}{\partial p_k} = \frac{c}{\pi^2 h^3}\, p_k^2\,(M_k^2 c^2 + p_k^2)^{1/2} + 3\,(\beta - \alpha)\, p_k^2 = 0, \tag{3.7}$$

where $k = \Sigma^-$ and Ξ^-,

$$\frac{\partial \Phi}{\partial p_k} = \frac{c}{\pi^2 h^3}\, p_k^2\,(M_k^2 c^2 + p_k^2)^{1/2} + 3\beta p_k^2 = 0, \tag{3.8}$$

where $k = n,\ n^*,\ \Lambda,\ \Sigma^\circ$ and Ξ°,

$$\frac{\partial \Phi}{\partial p_k} = \frac{c}{\pi^2 h^3}\, p_k^2\,(m_k^2 c^2 + p_k^2)^{1/2} - 3\alpha p_k^2 = 0 \tag{3.9}$$

where $k = e$ and μ, and, finally,

$$\frac{\partial \Phi}{\partial N_\pi} = m_\pi c^2 - 3\pi^2 h^3 \alpha = 0. \tag{3.10}$$

These equations, together with (3.3) and (3.4), completely determine the values of the threshold momenta p_k, and the parameters α and β. Eliminating α and β from (3.6)-(3.10), we obtain

$$(M_k^2 c^2 + p_k^2)^{1/2} = (M_n^2 c^2 + p_n^2)^{1/2} \tag{3.11}$$

where $k = n^*,\ \Lambda,\ \Sigma^\circ$, and Ξ°,

$$(M_k^2c^2 + p_k^2)^{1/2} = (M_n^2c^2+p_n^2)^{1/2} - (m^2c^2+p_e^2)^{1/2} \tag{3.12}$$

where k = p, p *, and Σ^+,

$$(M_k^2c^2 + p_k^2)^{1/2} = (M_n^2c^2 + p_n^2)^{1/2} + (m^2c^2 + p_e^2)^{1/2} \tag{3.13}$$

when k = Σ^- and Ξ^- and, finally,

$$(m_\mu^2c^2 + p_\mu^2)^{1/2} = (m^2c^2 + p_e^2)^{1/2} = m_\pi c. \tag{3.14}$$

Equations (3.11)-(3.14), together with (3.3) and (3.4), completely determine the particle concentrations N_k for the most general case, when the density of the gas consisting of leptons and baryons is so high that all of the possible elementary particles are present. It can be seen that, under conditions of thermodynamic equilibrium, the baryons in the same charge state possess equal threshold energies. This theorem also holds in the case of leptons.

Our problem is now to find the variation of the concentrations N_k as the total number of baryons N increases from zero to very high values. There will be a number of phase changes as N increases.

The First Phase

The baryon density is so low, that the sum of the proton and electron threshold energies is smaller than the rest mass of the neutron (and, hence, of all the other baryons). Only protons and electrons are present. In this case, we have the simple equation

$$N_p = N_e.$$

This will be called the proton-electron phase.

The Second Phase

The sum of the electron and proton threshold energies is greater than the neutron rest energy, while the electron threshold energy is lower than the μ-meson rest energy (neutron phase).

We then have

$$\begin{gathered}(M_p^2 c^2 + p_p^2)^{1/2} + (m^2 c^2 + p_e^2)^{1/2} = (M_n^2 c^2 + p_n^2)^{1/2} \\ N_e = N_p.\end{gathered} \tag{3.15}$$

This equation is obtained from (3.11)-(3.14) if the parameters of all the particles except the electron, proton, and neutron are omitted. Solving (3.15), we obtain

$$N_e = N_p = N_0 \chi^{-3} \{[1 + \alpha\chi/\pi + \chi^2 (N_n/N_0)^{2/3}]^{1/2} - 1\}, \tag{3.16}$$

where $\alpha \approx 2.54$ is the difference between the masses of the neutron and proton in units of electron mass, $\chi = 2\pi m/M_p = 3.39 \cdot 10^{-3}$, and $N_0 = 8(mc/h)^3 = 1.4 \cdot 10^{32}$ cm^{-3}.

The transition from the proton-electron to the neutron phase takes place at a baryon density of N = $0.77 \cdot 10^{31}$ cm^{-3}. The ratio of the number of protons to the number of neutrons decreases rapidly, and quickly becomes of the order of 10^{-3}.

The Third Phase

The electron threshold energy is higher than the rest energy of the μ^--meson, while the sum of the neutron and electron threshold energies is lower than the rest energy of the Σ^- (nucleon$-\mu^-$-meson) phase. From (3.11) to (3.14), we obtain, in this case,

$$\begin{gathered}(M_p^2c^2 + p_p^2)^{1/2} = (M_n^2c^2 + p_n^2)^{1/2} - p_e \\ (m_\mu^2c^2 + p_\mu^2)^{1/2} = p_e, \\ p_e^3 + p_\mu^3 = p_p^3.\end{gathered} \tag{3.17}$$

In this, as in the preceding phase, the electrons are highly relativistic. The number of μ^--mesons is here very small by comparison with the number of electrons, so that, for the calculation of the proton and electron concentrations, we can still use (3.16). The concentration of μ^--mesons is given by

$$N_\mu = N_e [1 - (A_\mu / N_e)^{2/3}]^{3/2}, \tag{3.18}$$

where $A_\mu = (1/3\pi^2)(m_\mu c/h)^3 = 5.24 \cdot 10^{36}$ cm^{-3} is the threshold for the creation of μ^--mesons. To this electron density there corresponds a neutron density approximately equal to $5 \cdot 10^{38}$ cm^{-3}, which is about two and a half times as great as the density of particles in normal nuclear matter.

The Fourth Phase

This phase appears when the density of matter is such that the sum of the neutron and electron threshold energies becomes equal to the rest energy of the Σ^--hyperon. The first Σ^--hyperons begin to appear at this density. As the density of matter increases further, the Λ, n^*, Σ°, Ξ^-, p^*, Σ^+ and Ξ° particles begin to appear in this order. This phase, which can be conveniently called the hyperon phase, consists of a number of subphases, each corresponding to the appearance of a new particle. However, we will not consider the properties of the individual subphases separately. From the general equations (3.11)-(3.14), omitting the π-meson rest energy (since this particle is still absent), we obtain

$$\begin{aligned} &E_\Lambda = E_{n^*} = E_{\Sigma^\circ} = E_{\Xi^\circ} = E_n \quad &(a)\\ &E_{\Sigma^+} = E_{p^*} = E_p = E_n - E_e \quad &(b)\\ &E_{\Sigma^-} = E_{\Xi^-} = E_n + E_e \quad &(c)\\ &E_e = E_\mu \quad &(d)\\ &p_p^3 + 2p_{p^*}^3 + p_{\Sigma^+}^3 - p_{\Sigma^-}^3 - p_{\Xi^-}^3 - p_\mu^3 - p_e^3 = 0, \quad &(e) \end{aligned} \tag{3.19}$$

where E_k is the threshold energy (chemical potential) of the particles.

From (3.19a) we obtain

$$N_k = \frac{1}{2} a_k N_n [1 - (A_k/N_n)^{2/3}]^{3/2}, \tag{3.20}$$

where the A_k are constants.

$$k = \Lambda,\ \Sigma^\circ,\ n^* \text{ and } \Xi^\circ,$$

$$A_k = \frac{1}{3\pi^2}\left(\frac{M_k c}{h}\right)^3 [1 - M_n^2/M_k^2]^{3/2}. \tag{3.21}$$

From (3.20) it can be seen that neutral hyperons can exist in the medium only at densities exceeding the neutron density $N_n > A_k$. Thus, the quantities A_k play the role of threshold densities for the corresponding particles. The numerical values of the threshold densities for the neutral hyperons are as follows:

$$A_k = \begin{array}{ll} 9.6 \cdot 10^{38} & \text{cm}^{-3} \text{ for } \Lambda \\ \sim 9.6 \cdot 10^{38} & \text{cm}^{-3} \text{ for } n^* \\ 1.72 \cdot 10^{39} & \text{cm}^{-3} \text{ for } \Sigma^\circ \\ 3.55 \cdot 10^{39} & \text{cm}^{-3} \text{ for } \Xi^\circ. \end{array} \tag{3.21'}$$

By way of example, we note that, at a neutron density $N_n = 4 \cdot 10^{39}$ cm^{-3}, the densities of the Λ-, Σ-, and Ξ°- hyperons are $1.92 \cdot 10^{39}$, $1.12 \cdot 10^{39}$, and $8.2 \cdot 10^{37}$ cm^{-3}, respectively, while the number of particles in the neutron-isobar state is $3.85 \cdot 10^{39}$ cm^{-3}. When the density of matter is 10^{40} cm^{-3}, the concentration of all neutral particles will be of the same order of magnitude.

With the help of (3.19b), the densities of all positive particles can be expressed in terms of the proton density:

$$N_k = \frac{1}{2} a_k N_p [1 - (B_k/N_p)^{2/3}]^{3/2}; \quad k = p^* \text{ and } \Sigma^+, \tag{3.22}$$

where B_k is given by

$$B_k = \frac{1}{3\pi^2}\left(\frac{M_k c}{h}\right)^3 \left(1 - \frac{M_p^2}{M_k^2}\right)^{3/2}. \tag{3.23}$$

As in the case of the neutral hyperons, the quantities B_k represent the threshold densities for positive hyperons. They can be present in the medium only for proton densities $N_p > B_k$. The numerical values of these constants are

$$B_k = \begin{matrix} 1.73\cdot 10^{39}\ \mathrm{cm}^{-3}\ \text{for}\ \Sigma^+ \\ \sim 0.98\cdot 10^{39}\ \mathrm{cm}^{-3}\ \text{for}\ p^* \end{matrix} \qquad (3.24)$$

In an analogous way, from (3.19c), we find that

$$N_{\Xi^-} = N_{\Sigma^-}\,[1 - (B_{\Xi^-}/N_{\Sigma^-})^{2/3}]^{3/2}, \qquad (3.25)$$

where

$$B_{\Xi^-} = \frac{1}{3\pi^2}\left(\frac{M_{\Xi^-}c}{h}\right)^3 (1 - M_{\Sigma^-}^2/M_{\Xi^-}^2)^{3/2} = 7.87\cdot 10^{38}\ \mathrm{cm}^{-3}.$$

Thus, for the complete solution of the problem of determining the particle concentrations at a given density of matter, we have only to express the electron, proton, and Σ^--hyperon densities in terms of the neutron density. Instead of the total density of matter, we prescribe the neutron density, and this is equivalent to the specification of the constant in (3.4). We are left with three unused equations: (3.3), one of the equations (3.19b), and one of the equations (3.19c) and (3.19d) for the three unknowns N_e, N_p, and N_{Σ^-}. These equations can be rewritten as

$$p_p^3\left[1 + 2\left(1 - \frac{b_{p^*}^2}{p_p^2}\right)^{3/2} + \left(1 - \frac{b_{\Sigma^+}^2}{p_p^2}\right)^{3/2}\right] - p_{\Sigma^-}^3\left[1 + \left(1 - \frac{b_{\Xi^-}^2}{p_{\Sigma^-}^2}\right)^{3/2}\right] - p_e^3\left[1 + \left(1 - \frac{m_\mu^2 c^2}{p_e^2}\right)^{3/2}\right] = 0, \qquad (3.26)$$

$$cp_p = [(E_n - E_e)^2 - M_p^2c^4]^{1/2}; \quad cp_{\Sigma^-} = [(E_n + E_e)^2 - M_{\Sigma^-}^2c^2]^{1/2},$$

where

$$b_k = (3\pi^2)^{1/3}hB_k^{1/3} = \begin{matrix} \sim 3.23\cdot 10^{-14} & \text{for}\ p^* \\ 3.9\ \cdot 10^{-14} & \text{for}\ \Sigma^+ \\ 3.0\ \cdot 10^{-14} & \text{for}\ \Xi^-. \end{matrix} \qquad (3.27)$$

The constants b_k are related to the masses of the corresponding particles by the simple relation $b_k = c(M_k^2 - M_l^2)^{\frac{1}{2}}$, where the subscript l stands for p in the case of positive particles, and for Σ^- in the case of the Ξ^- - hyperon. Equations (3.22), (3.18), and (3.25) have been taken into account in the derivation of the first of (3.26).

Equations (3.26) have been solved graphically. The results are given in Figs. 1 and 2. Figure 1 gives the electron density as a function of the neutron density. For $N_n < 5\cdot 10^{38}\ \mathrm{cm}^{-3}$, there are no μ^--mesons or hyperons in the medium, and in this region $N_e = N_p$. For $N_n < 10^{35}\ \mathrm{cm}^{-3}$, the electron (or proton) concentration increases slowly with N_n, and is three to four orders of magnitude smaller than the neutron concentration. With a further increase of N_n (when $N_n > 10^{35}\ \mathrm{cm}^{-3}$), $\log N_e$ increases more rapidly. Most of the electrons are highly relativistic. At $N_n > 5\cdot 10^{38}\ \mathrm{cm}^{-3}$, μ^--mesons begin to appear, although they are few in number, and $N_e \approx N_p$. Σ^--hyperons appear at a certain threshold value of N_n. This corresponds to a threshold momentum p_n, given by

$$\{[(M_{\Sigma^-}c^2 - E_n)^2 - m_\mu^2c^4]^{3/2} + (M_{\Sigma^-}c^2 - E_n)^3\}^2 = [(2E_n - M_{\Sigma^-}c^2)^2 - M_p^2c^4]^3. \qquad (3.28)$$

The solution of this equation gives $p_n = 517$ Mev/c, from which we obtain $N_n = 6.13\cdot 10^{38}\ \mathrm{cm}^{-3}$.

It is remarkable that, at this density, there are still no Λ-hyperons which, as was shown above, appear at a threshold baryon density of $1.25\cdot 10^{39}\ \mathrm{cm}^{-3}$. This occurs despite the fact that the rest energy of the Σ^--particles is considerably greater than that of the Λ. The reason for this is that the Σ^--particles have to neutralize the positive charge of the protons whose concentration increases with increasing N_n, while, starting from a certain point, the Σ^--particles on energy grounds are more "economical" than one new proton and two new electrons. As the density increases further, Σ^0-hyperons, Ξ^--hyperons, then Σ^+-hyperons and Ξ^0-hyperons appear successively after the Λ-hyperons. It is interesting to note that the negative hyperon Ξ^- again appears before the lighter Σ^+-hyperon. All of these threshold densities are found to lie below the threshold for the appearance of π^--mesons. The values of the threshold baryon concentrations and the total density of matter at which different hyperons appear are given in the table below.

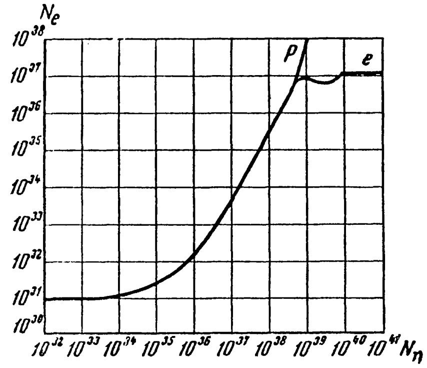

Fig. 1. The electron density N_e as a function of the neutron density N_n. Scales are logarithmic. Ordinates of the curve for $N_n < 6 \cdot 10^{38}$ cm^{-3} also give proton density. After this critical point, the proton curve suffers a change in slope and becomes very steep, while the electron curve at first dips appreciably and then begins to rise. This decrease of N_e is governed by the appearance and subsequent rapid increase in the concentration of Σ^--hyperons. For $N_n \geq 8.5 \cdot 10^{39}$ cm^{-3}, the electron concentration remains constant, which is due to the presence of a π^--meson gas.

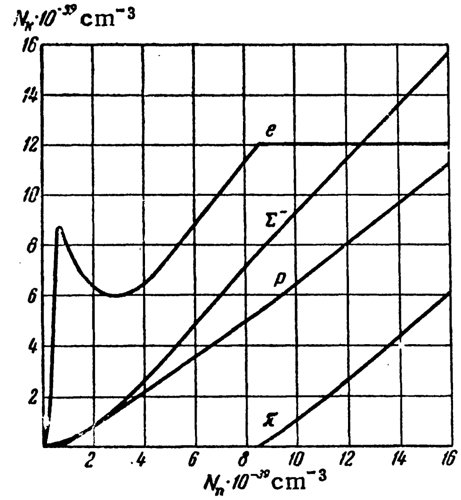

Fig. 2. The proton, Σ^--hyperon, π^--meson, and electron densities as a function of the neutron density. The ordinates of the curve for electrons have been multiplied by 10^3. There is a small discontinuity in all of the curves at $N_n = 8.5 \cdot 10^{38}$ cm^{-3}, because of the creation of π^--mesons.

The Values of Some Parameters Characterizing the Thresholds for the Creation of Different Particles

Particle	n	μ^-	Σ^-	Λ	n^*	Σ^0	Ξ^-	p^*	Σ^+	Ξ^0	π^-
t_n	0	1.92	2.10	2.41	2.41	2.856	2.97	3.07	3.42	3.49	4.36
Baryon density $N \cdot 10^{-39}$ cm^{-3}	$7.68 \cdot 10^{-9}$	0.465	0.640	1.27	1.27	3.95	5.10	6.44	13.5	15.4	58.6
Density of matter, $\rho \cdot 10^{-15}$ g·cm^{-3}	$1.28 \cdot 10^{-3}$	0.813	1.12	2.36	2.36	7.82	10.3	13.2	28.8	33.0	144

Remark. t_n is given by (4.4).

Figure 2 shows the electron, proton, and Σ^--hyperon densities as a function of the neutron density. These curves, together with (3.22) and (3.25), give the density of all charged baryons for a given value of the neutron density.

The Fifth Phase

Here, the threshold energy of the electrons and μ^--mesons becomes equal to the rest energy of the π^--meson. π^--Mesons appear from this stage onwards. In this case, all of the equations of the system (3.11)-(3.14) are valid. With the appearance of π^--mesons, the concentrations of electrons and μ^--mesons remain constant, while the

π^--meson concentration increases very rapidly with increasing baryon density, and quickly becomes of the same order of magnitude as the concentration of each type of baryon, separately. The threshold for production of π^--mesons can be easily found from a comparison of the equations mentioned above, or of (3.14) and Fig. 2. It is found to be $N_e = 1.20 \cdot 10^{37}$ cm^{-3}, or, in terms of the neutron and baryon densities, $N_n = 8.5 \cdot 10^{39}$ cm^{-3} and $N = 5.8 \cdot 10^{40}$ cm^{-3}, respectively.

From (3.14), we find the concentrations of electrons and μ^--mesons to be

$$N_e = \frac{1}{3\pi^2}\left(\frac{m_\pi c}{h}\right)^3 = 1.20 \cdot 10^{37}\ \text{cm}^{-3}$$
$$N_\mu = \frac{c^3}{3\pi^2}(m_\pi^2 - m_\mu^2)^{3/2}/h^3 = 3.35 \cdot 10^{38}\ \text{cm}^{-3} \qquad (3.29)$$
$$\text{for} \quad N_n > 8.5 \cdot 10^{39}\ \text{cm}^{-3}$$

Further, (3.20), (3.22), and (3.25), describing the relation between the concentrations of baryons of equal charge, remain in force for this phase. We can now also find analytically the relation between the concentrations of charged and neutral hyperons. Thus, from (3.12) and (3.14), we find that

$$N_k = \frac{1}{2} a_k \{[(C_n^{2/3} + N_n^{2/3})^{1/2} - C_\pi^{1/3}]^2 - C_k^{2/3}\}^{3/2},$$
$$C_k = \lambdabar_k^{-3}/3\pi^2, \qquad (3.30)$$

where $\lambdabar = h/M_k c$ is the Compton wavelength for the kth particle, divided by 2π, while the index k stands for p, p*, and Σ^+.

Similarly, from (3.13) and (3.14) for the concentrations of negative hyperons, we find that

$$N_k = \{[(C_n^{2/3} + N_n^{2/3})^{1/2} + C_\pi^{1/3}]^2 - C_k^{2/3}\}^{3/2}. \qquad (3.31)$$

The notation is the same as that used in (3.30). The constants C_k have the following values:

$$C_k = \begin{cases} 1.20 \cdot 10^{37} & \text{for } \pi\text{- particle} \\ 3.66 \cdot 10^{39} & \text{for } p \quad " \\ 3.67 \cdot 10^{39} & \text{for } n \quad " \\ 6.14 \cdot 10^{39} & \text{for } p^* \quad " \\ 7.43 \cdot 10^{39} & \text{for } \Sigma^+ \quad " \\ 7.57 \cdot 10^{39} & \text{for } \Sigma^- \quad " \\ 10.2 \cdot 10^{39} & \text{for } \Xi^- \quad " \end{cases} \qquad (3.32)$$

Hence, the concentration of all particles, with the exception of π^--mesons, have been expressed as functions of N_n. The concentration of π^--mesons can be calculated from the equation

$$N_\pi = N^+ - N^- - N_e - N_\mu, \qquad (3.3')$$

where N^+ and N^- are the total concentration of the positive and negative baryons, respectively. The results are given in Fig. 2. Small changes in the slopes of the curves for electrons, protons, and Σ^--hyperons at $N_n = 8.5 \cdot 10^{39}$ cm^{-3} are the result of the appearance of π^--mesons.

The relative concentration of hyperons as a function of the concentration of baryons is shown in Fig. 3.

In the extremely relativistic case, i.e., at very high densities of matter which may not occur in nature, we have the following asymptotic relations:

$$N_{\Sigma^+} \simeq N_p \simeq 0.5\, N_{p^*} \simeq N_{\Xi^-} \simeq N_{\Sigma^-} \approx N_\Lambda \approx 0.5\, N_{n^*} \approx N_{\Sigma^0} \approx N_{\Xi^0} \approx N_n,$$
$$N_\pi \approx N^+ - N^- \approx 2N_n. \qquad (3.33)$$

Thus, on the basis of the results obtained in this section, we come to the conclusion that, at densities of matter $N \gtrsim 10^{39}$ baryons cm^{-3}, the medium, in addition to nucleons, also contains hyperons and excited nucleons. The concentrations of all baryons are of the same order of magnitude, while those of electrons and μ^--mesons are three orders of magnitude lower. For baryon densities, $N > 6 \cdot 10^{40}$ cm^{-3}, π^--mesons also appear in comparatively large numbers.

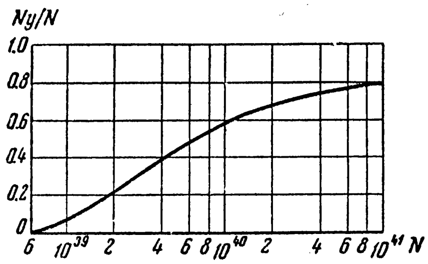

Fig. 3. The baryon density is plotted along the abscissa, while the ordinate is the ratio of the density of hyperons to that of all baryons.

For the approach to the problem considered in this section, it is not necessary to have T = 0. It is clear that the results obtained above will remain valid for temperatures different from zero, so long as the temperature is such that the baryon gas can be assumed to be highly degenerate. On the other hand, since the density of matter is very high, the temperatures allowed in this case can, nevertheless, be very high.

The critical density at which hyperons appear in the medium is approximately three times as high as the normal nuclear density. The question now arises whether any forces and, in particular, the forces arising between baryons at very small distances (incompressibility), can prevent the formation of states with such high densities. To answer this question, let us consider the differences between the physical conditions existing in normal atomic nuclei and in the nuclear matter investigated by us. In contrast to normal nuclei, the nuclear matter in stars is neutral and, moreover, it is situated in a strong hydrostatic pressure created by gravitational forces. Obviously, both of these features create conditions favorable for a further increase of the density. The matter under these physical conditions would be compressible without limit, if there were no repulsive forces acting between nucleons, at distances of the order of $0.4 \cdot 10^{-13}$ cm. These forces are so strong, that they are usually approximated by a Dirac δ-function, i.e., the nucleon is assumed to have an impenetrable core with a radius of the order of $2 \cdot 10^{-14}$ cm [12]. It is highly probable that these repulsive forces will, in the end, balance the external hydrostatic pressure, and the compression of matter to a density greater than a limiting value will be prevented. This limiting density will apparently be greater than $N \sim (0.4 \cdot 10^{-13})^{-3} = 1.6 \cdot 10^{40}$ cm^{-3}, i.e., above our threshold for the appearance of hyperons, but lower than the threshold for the appearance of π^--mesons. Thus, it appears that physical conditions favorable to the appearance of hyperons as stable particles can be realized in nature. On the other hand, the problem of the appearance of π^--mesons at high densities may require review, since the presence of repulsive forces may invalidate a theory in which such interactions are not taken into account.

The region of a star which, in addition to nucleons, also contains hyperons, we will call the "hyperon core." It must be surrounded by a spherical layer mainly consisting of neutrons. There are no hyperons in this layer, protons and electrons are present in equal numbers, while their concentration is approximately three orders of magnitude lower than that of neutrons. We will call this region of the star the "neutron layer." Outside this layer, there is a region in which matter consists of electrons, protons, and other nuclei. In the deeper layers of this region, the atoms are completely ionized. In the following, we will call this region of the star the "outer envelope."

4. THE EQUATION OF STATE

Up to now, we have considered the equilibrium composition of matter with a given density. However, the question of what values of density are attained in any given cosmic body which is in equilibrium under the action of its own gravitational forces can only be answered after we have constructed a model for the equilibrium configuration of this body, when the attraction is balanced by pressure. To do this, we have to use Einstein's theory of gravitation, in view of the very high density.

Einstein's equations contain the energy-momentum tensor whose components are determined by the proper density of matter and the proper pressure, i.e., the density and pressure measured by an observer situated at the given point. A solution of Einstein's equation becomes possible only when the pressure is given as a function of the density, i.e., when the equation of state is known. In the present paper, we will restrict ourselves to the derivation of the equation of state for a degenerate baryon gas with the high densities considered above.

Let us consider a small volume V inside which the gravitational field and, consequently, the concentrations of the particles, can be assumed constant. The energy contained in this volume is given by

$$E = V\left\{\sum_k\left[\int_0^{p_k} E_k(p)\,dN_k(p) + N_k U(N)\right] + N_\pi m_\pi c^2\right\}, \tag{4.1}$$

where $E_k = c(M_k^2c^2 + p_k^2)^{\frac{1}{2}}$ is the energy of the kth particle, $dN_k(p)$ is the number of these particles with momenta lying in the interval (p, p + dp), U(N) is the nuclear potential energy per particle which is assumed to be the same for all baryons, and $N = \Sigma_k N_k$ is the total baryon density. The energy of the electron gas is low, and will be neglected. The reason for writing the energy in the form of (4.1) is that the energy-momentum tensor T_{ik} appearing in Einstein's equation must be taken as the sum of the energy-momentum tensors of matter and the nuclear field.

In the region of normal nuclear densities $10^{38} \lesssim N \lesssim 10^{39}$ cm^{-3}, the energy U(N) is sufficiently small by comparison with E_k, and may be neglected. In fact, the average kinetic energy of the nucleons in nuclei is approximately equal to 27 Mev, while the binding energy, corrected for the absence of repulsive Coulomb forces and surface effects, is equal to 15 Mev. Consequently, the depth of the potential well is equal to 42 Mev. It is obvious that U(N) is not a constant, but depends on the density of particles N. In the region of densities $10^{37} \lesssim N \lesssim 10^{39}$ cm^{-3}, this function can be obtained from a comparison of Figs. 10 and 11 of [11]. The algebraic sum of the ordinates of these curves is equivalent to our single-particle potential energy U(N). Thus, as N increases, U(N) at first decreases, reaching a minimum of 40 Mev at $N \approx 4 \cdot 10^{38}$ cm^{-3}, and then, for $N > 4.5 \cdot 10^{38}$ cm^{-3}, i.e., at densities only slightly lower than the threshold value for the production of Σ^--hyperons, it begins to increase rapidly. The minimum of U(N) occurs when the average distance between particles is of the order of the π-meson Compton wavelength $h/m_\pi c$. In the paper referred to, it has been assumed that, at a certain distance between particles (0.4 fermis), the potential energy increases discontinuously to infinity, which corresponds to the assumption of an impenetrable nucleon core.

In reality, of course, the nucleon core is not ideally rigid, and the infinitely large repulsive force acting at a fixed separation between particles must be replaced by a more realistic model of the interaction. This is necessary, since the expression for the pressure contains the derivative dU/dN, which is equivalent to a specification of the repulsive force. In other words, for $N > 10^{39}$ cm^{-3}, the behavior of the function U(N) will be very important, and the use of a δ-function as an approximation to it is completely inadequate.

Because, in the following, we will also have to consider problems associated with very high densities, we found it desirable to retain in (4.1) the term containing U(N). It should be noted, however, that the representation of the potential energy of one particle by U(N) is itself a rough approximation, because the actual interaction energy can also depend on the distribution of the particle momenta, as well as on other parameters.

After integration, (4.1) yields the following expression for the energy density:

$$\rho = \frac{c}{16\pi^2 h^3} \sum_k a_k \Bigg[p_k (M_k^2 c^2 + 2p_k^2)(M_k^2 c^2 + p_k^2)^{1/2} - M_k^4 c^4 \ln \frac{p_k + \sqrt{M_k^2 c^2 + p_k^2}}{M_k c} \Bigg] + NU(N) + N_\pi m_\pi c^2. \tag{4.2}$$

Further, the derivative of (4.1) with respect to the volume gives the pressure with opposite sign:

$$P = \frac{c}{48\pi^2 h^3} \sum_k a_k \Bigg[p_k (2p_k^2 - 3M_k^2 c^2)(M_k^2 c^2 + p_k^2)^{1/2} + 3M_k^4 c^4 \ln \frac{p_k + \sqrt{M_k^2 c^2 + p_k^2}}{M_k c} \Bigg] + N^2 \frac{dU(N)}{dN}. \tag{4.3}$$

The partial pressure of electrons and μ-mesons has been neglected. At $N \approx 4 \cdot 10^{38}$ cm^{-3}, the derivative $dU/dN \approx \approx 0$, but with increasing values of N, it rapidly increases. For $N < 4 \cdot 10^{38}$ cm^{-3}, nuclear forces, to some extent, lower the internal gas pressure ($dU/dN < 0$); while, for $N > 4 \cdot 10^{38}$ cm^{-3}, they increase the pressure ($dU/dN > 0$). Thus, with sufficiently high values of dU/dN, the internal pressure may become large enough to balance the gravitational forces, and this will affect the compressibility of the star. In all probability, this will occur at particle concentrations of the order of $2 \cdot 10^{40}$ cm^{-3}.

Equations (4.2) and (4.3), together, determine the equation of state. For further applications, it is convenient to replace the threshold momenta p_k by the parameters [6, 13, 14]

$$t_k = 4 \ln \frac{p_k + \sqrt{M_k^2 c^2 + p_k^2}}{M_k c}. \tag{4.4}$$

Eliminating p_k from (4.2) and (4.3), we obtain the equation of state in parametric form:

$$\rho = \frac{1}{2} K_n \sum_k a_k \left(\frac{M_k}{M_n}\right)^4 (sht_k - t_k) + NU(N) + N_\pi m_\pi c^2 \qquad (4.5)$$

$$P = \frac{1}{6} K_n \sum_k a_k \left(\frac{M_k}{M_n}\right)^4 \left(sht_k - 8sh\frac{t_k}{2} + 3t_k\right) + N^2 \frac{dU}{dN}, \qquad (4.6)$$

where

$$K_n = M_n^4 c^5 / 32\pi^2 h^3.$$

The particle density N can also be expressed in terms of t_k:

$$N = \frac{16}{3} \frac{K_n}{M_n c^2} \sum_k a_k \left(\frac{M_k}{M_n} sh \frac{t_k}{4}\right)^3. \qquad (4.7)$$

With the help of (3.20), (3.22), and (3.25), as well as Fig. 2, it is possible to express all of the parameters t_k in terms of the corresponding parameter for neutrons t_n.

CONCLUSIONS

The investigation of the properties of a neutral degenerate gas consisting of elementary particles at T = 0, leads to the following conclusions.

1. At a density $\rho < \rho_n$, where $\rho_n = 1.28 \cdot 10^7$ g · cm^{-3}, the gas consists of protons and electrons.

2. At $\rho = \rho_n$, neutrons appear. As the density increases further, the number of protons increases much more slowly than the number of neutrons. At densities higher than $2 \cdot 10^8$, the number of neutrons is already very much larger than the numbers of protons and electrons. At these densities, the degenerate gas can be considered to be practically a neutron gas.

3. At $\rho = \rho_{\Sigma^-} = 1.1 \cdot 10^{15}$ g · cm^{-3}, the first hyperons appear. In spite of the fact that the Λ-, Σ^{+-}, and Σ^0-particles have rest masses smaller than that of the Σ^-, the first to appear are Σ^--particles. As the density continues to increase up to a value $\rho = \rho_\Lambda$, where $\rho_\Lambda = 2.36 \cdot 10^{15}$ g · cm^{-3}, the number of Σ^--hyperons continues to grow, although hyperons of other types as yet do not appear.

4. At $\rho = \rho_\Lambda$, Λ-hyperons appear, and with a further increase of the density, other heavier hyperons make an appearance. Thus, at densities of the order of 10^{16} g · cm^{-3}, we have a baryon gas which is a mixture of nucleons, hyperons, and nucleon isobars, the concentrations of the different types of baryons being of the same order or magnitude.

At baryon densities exceeding $2 \cdot 10^{40}$ cm^{-3} ($5 \cdot 10^{16}$ g · cm^{-3}), the investigation of the state of such a gas encounters the following difficulties.

a. Because of the small distances between baryons, very strong repulsive forces arise, whose properties are not established exactly at the present time.

b. The relative concentrations of the various types of baryons can be strongly affected by the presence of higher hyperons (having masses greater than that of the Ξ^- particle). Therefore, any conclusions obtained for this range of densities would be premature. It is only possible to say that the relative concentration of the higher hyperons will increase with increasing density, while, at a certain value of the density, the existence of π^--mesons as a Bose gas becomes possible.

All that has been said above leads us to conclude that the model of superdense stars as purely neutron, or almost purely neutron bodies, should be replaced by a more complicated model, according to which a superdense star has a hyperon core, a neutron layer around the core, and a proton-electron envelope. As will be shown in a subsequent paper, the largest fraction of the mass (for a sufficiently large M) is concentrated in the hyperon core.

Finally, let us note that, at densities considerably smaller than nuclear densities, nuclei of the common atoms may exist in the medium. Thus, if the fusion of nucleons into α-particles is taken into account, it will be found that there are no protons at equilibrium. However, at the high densities considered by us, the individual atomic nuclei will no longer have an important role.

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